LCC Converter Small Signal Modeling

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Introduction

- Demanding efficiency requirements are driving engineers to the LLC resonant converter

80 PLUS Certification	115 V Internal Non-Redundant				230 V Internal Redundant			
% of Rated Load	**10%**	**20%**	**50%**	**100%**	**10%**	**20%**	**50%**	**100%**
80 PLUS	–	80%	80%	80%	N/A			
80 PLUS Bronze	–	82%	85%	82%	–	81%	85%	81%
80 PLUS Silver	–	85%	88%	85%	–	85%	89%	85%
80 PLUS Gold	–	87%	90%	87%	–	88%	92%	88%
80 PLUS Platinum	–	90%	92%	89%	–	90%	94%	91%
80 PLUS Titanium	–	–	–	–	90%	94%	**96%**	91%

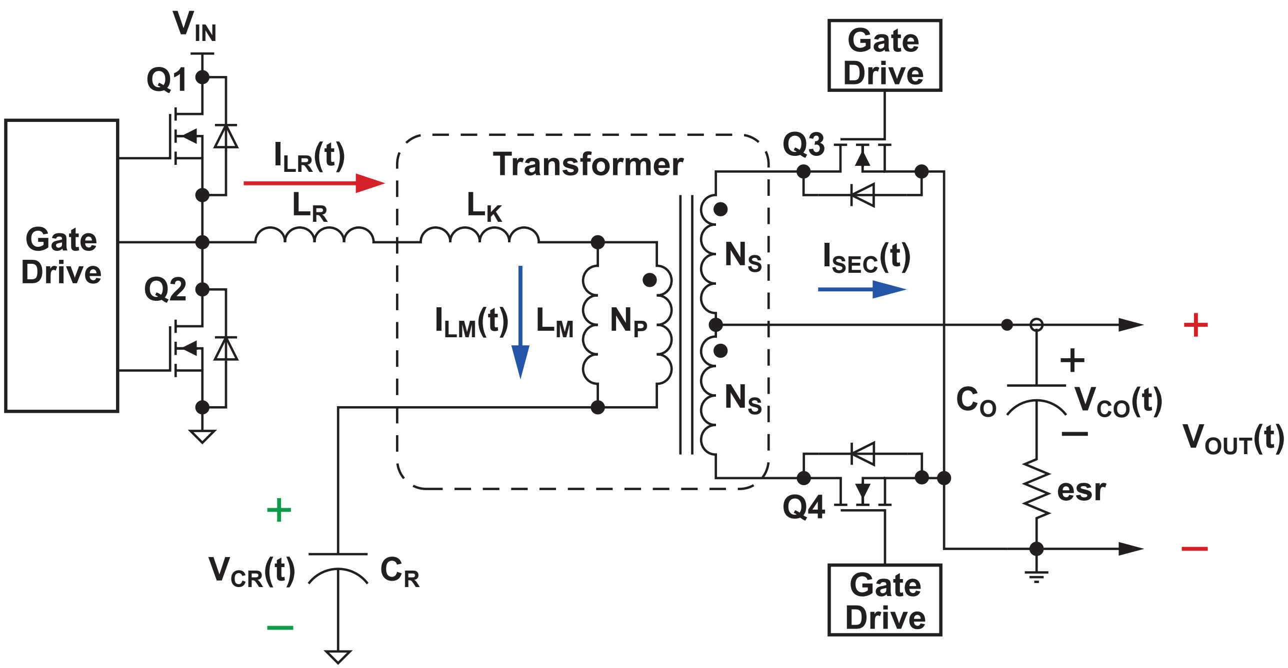
Courtesy: http://www.plugloadsolutions.com/80PlusPowerSupplies.aspx

- How do we identify and verify a robust set of compensation values for this converter?

Discussion Outline

- LLC Converter
- Modeling Process
- Case Study
- Tools
- Practical Limitations
- Conclusion

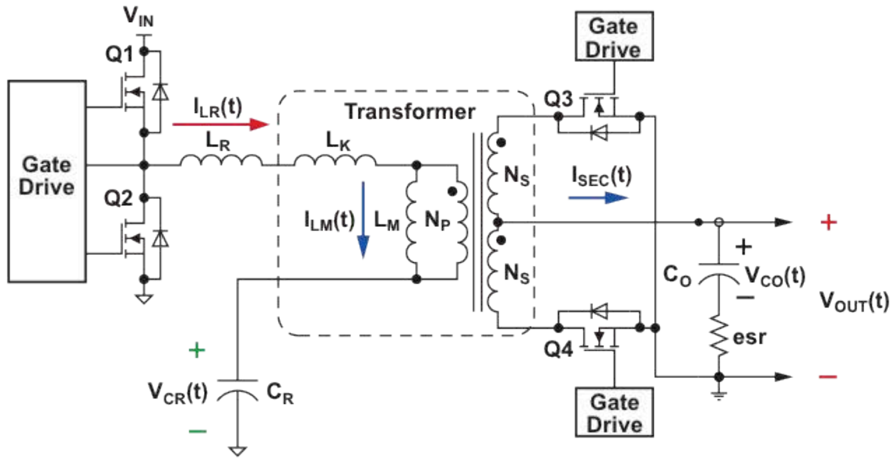
Operating States

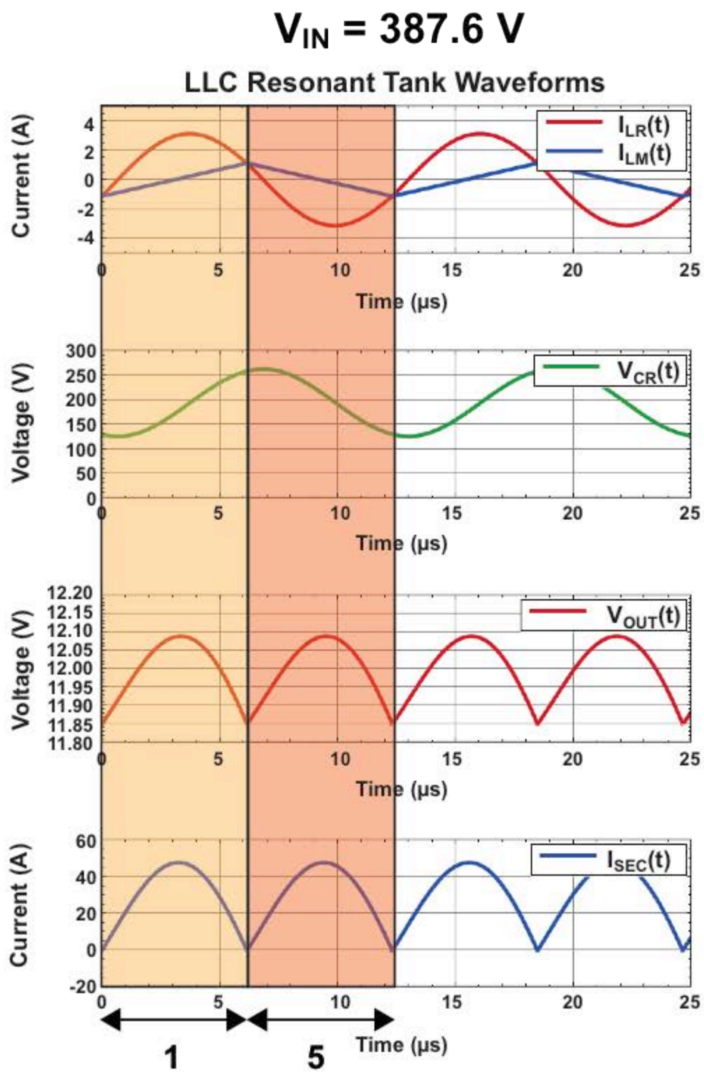


State Variables
$I_{LR}(t)$
$I_{LM}(t)$
$V_{CR}(t)$
$V_{CO}(t)$

State	Q1	Q2	Q3	Q4
1	ON	OFF	OFF	ON
2	ON	OFF	ON	OFF
3	ON	OFF	OFF	OFF
4	OFF	ON	OFF	ON
5	OFF	ON	ON	OFF
6	OFF	ON	OFF	OFF

Mode: Resonance

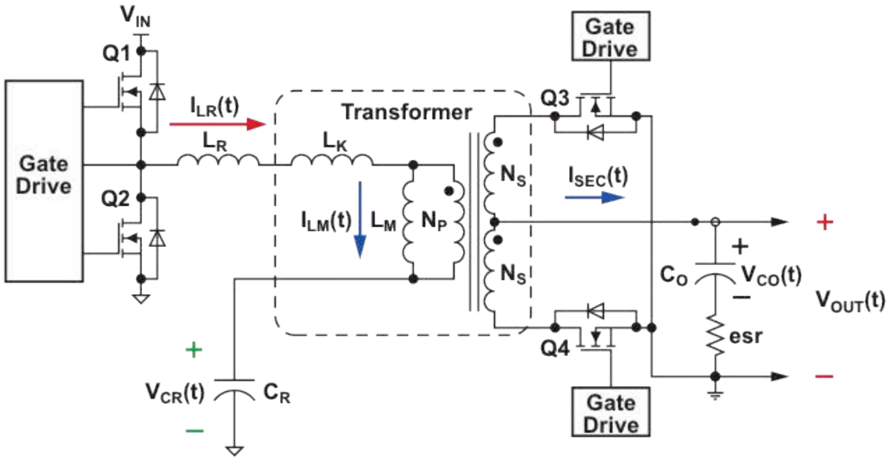


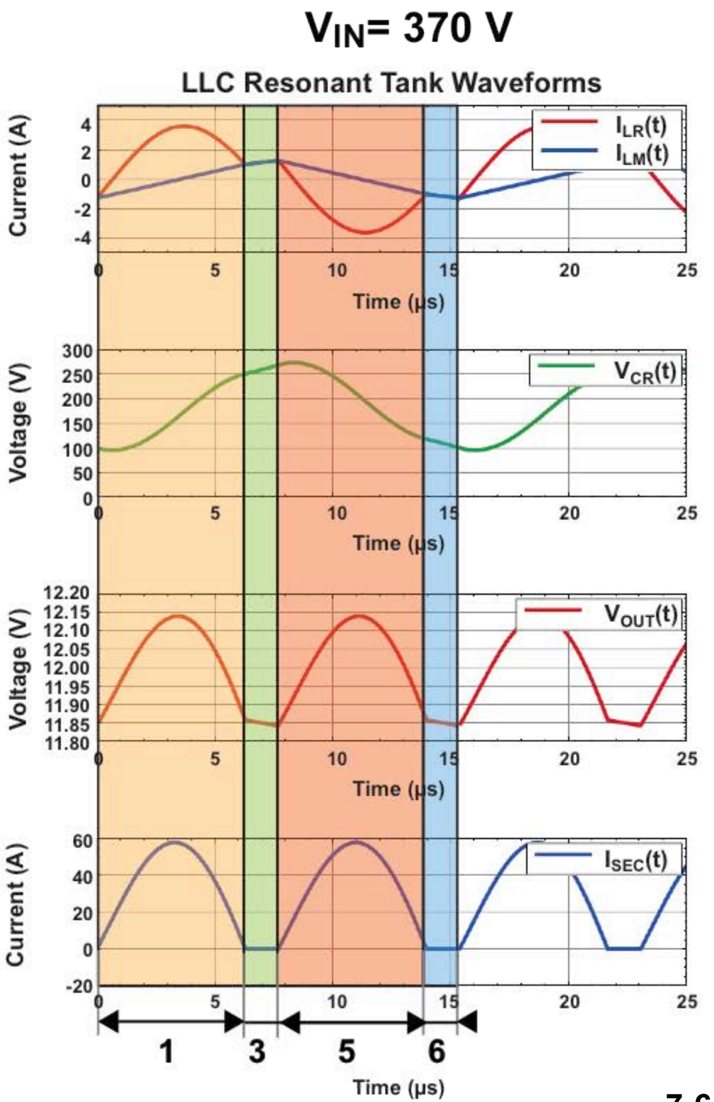


Mode State Sequence: 1→5

State	Q1	Q2	Q3	Q4
1	ON	OFF	OFF	ON
2	ON	OFF	ON	OFF
3	ON	OFF	OFF	OFF
4	OFF	ON	OFF	ON
5	OFF	ON	ON	OFF
6	OFF	ON	OFF	OFF

Mode: Below Resonance

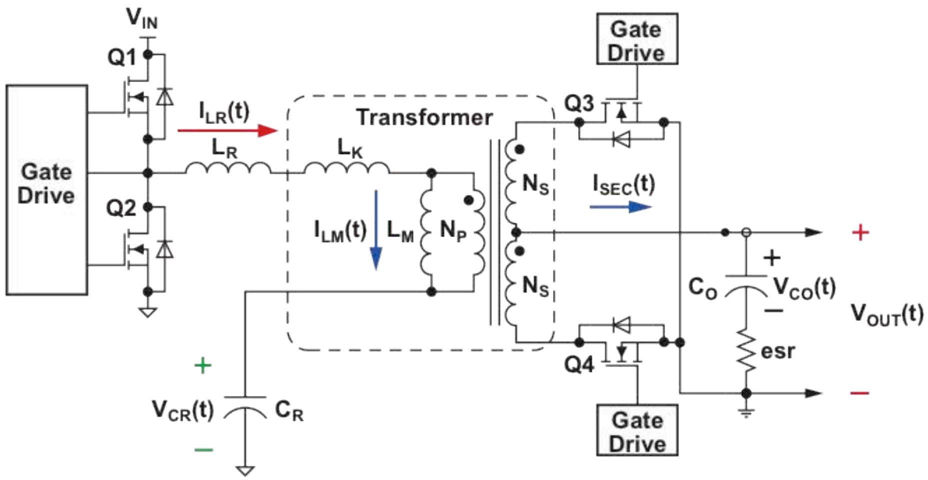


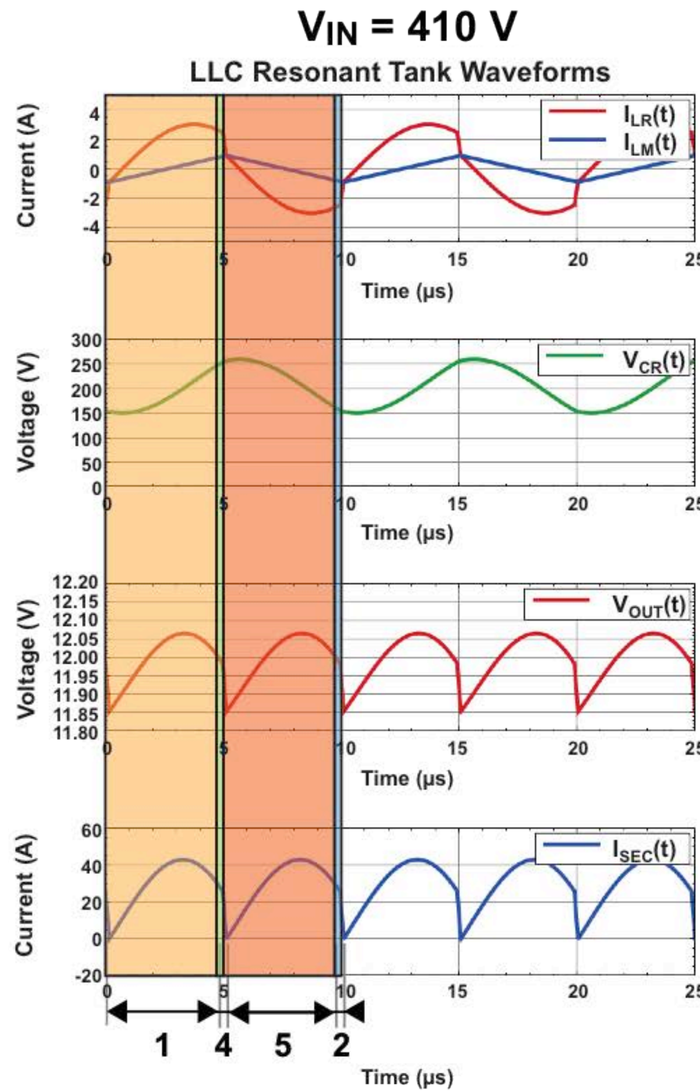


Mode State Sequence: 1→3→5→6

State	Q1	Q2	Q3	Q4
1	ON	OFF	OFF	ON
2	ON	OFF	ON	OFF
3	ON	OFF	OFF	OFF
4	OFF	ON	OFF	ON
5	OFF	ON	ON	OFF
6	OFF	ON	OFF	OFF

Mode: Resonance





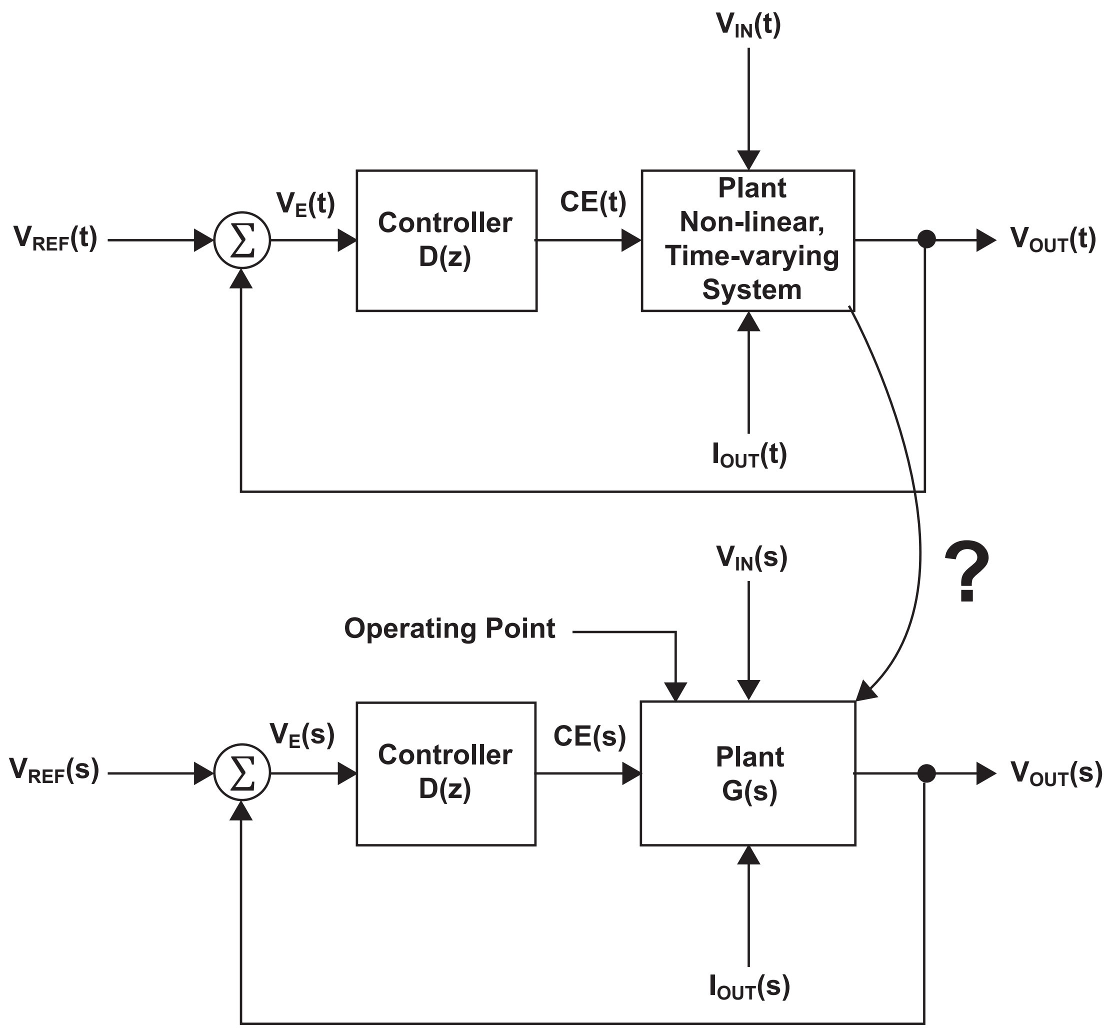
Mode State Sequence: 1→4→5→2

State	Q1	Q2	Q3	Q4
1	ON	OFF	OFF	ON
2	ON	OFF	ON	OFF
3	ON	OFF	OFF	OFF
4	OFF	ON	OFF	ON
5	OFF	ON	ON	OFF
6	OFF	ON	OFF	OFF

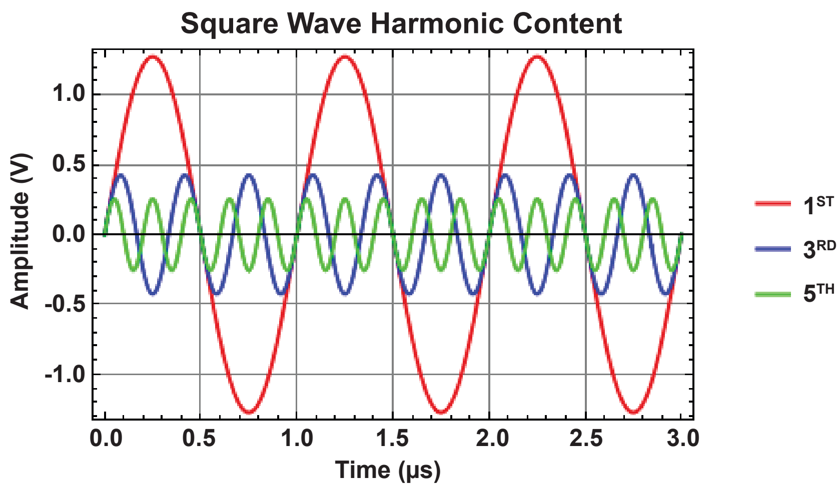
Modeling Process Overview

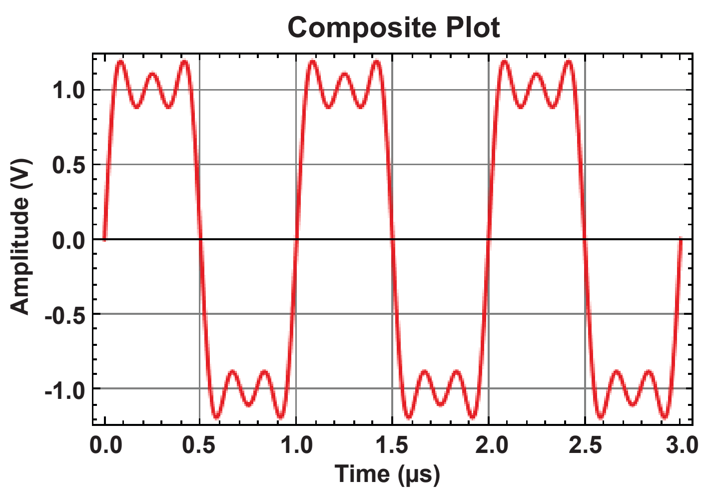
Fundamental Steps

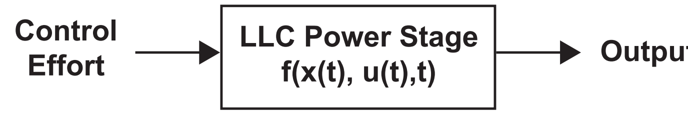
1. Identify the states and modes used
2. Average the states for the identified mode
3. Calculate DC operating point
4. Linearize the result

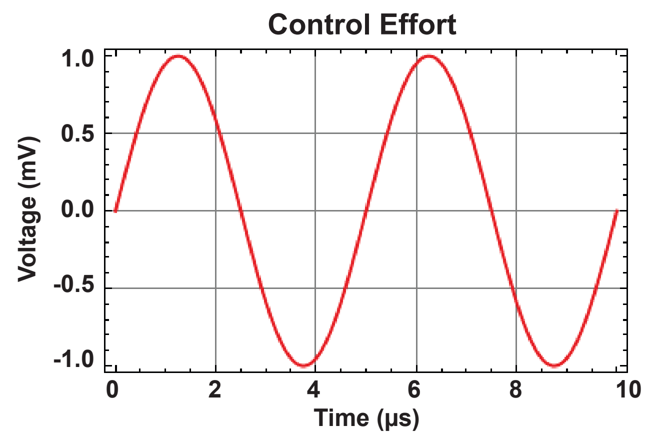


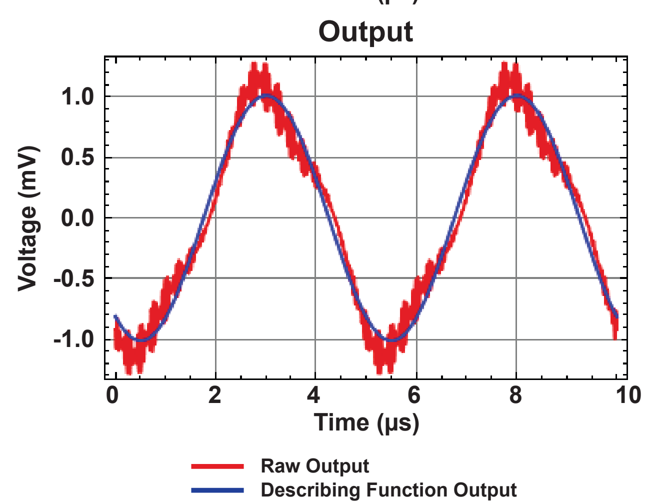
Fourier Series



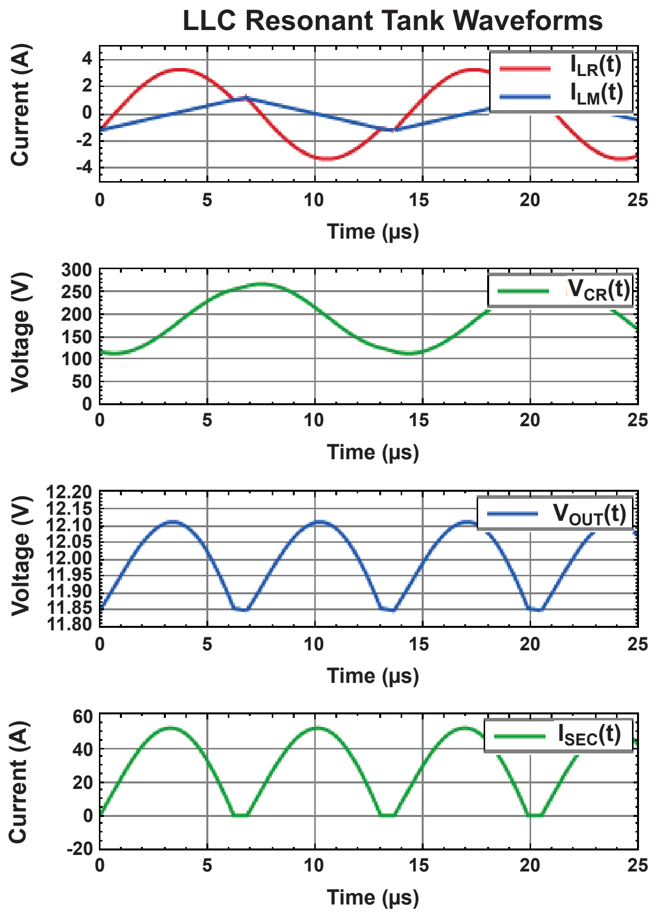


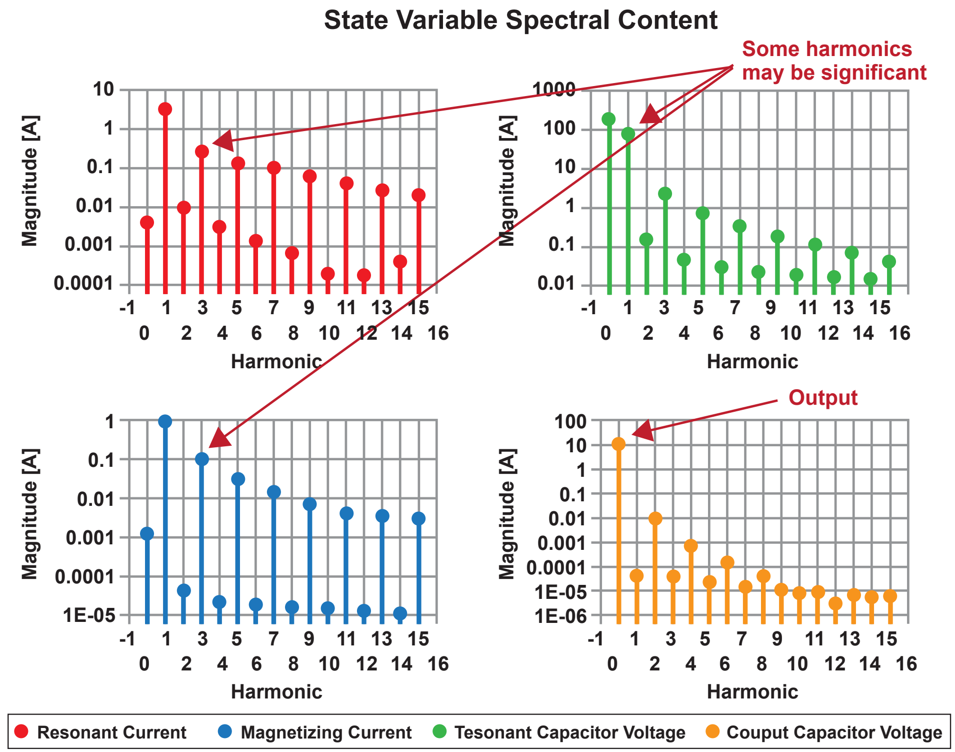






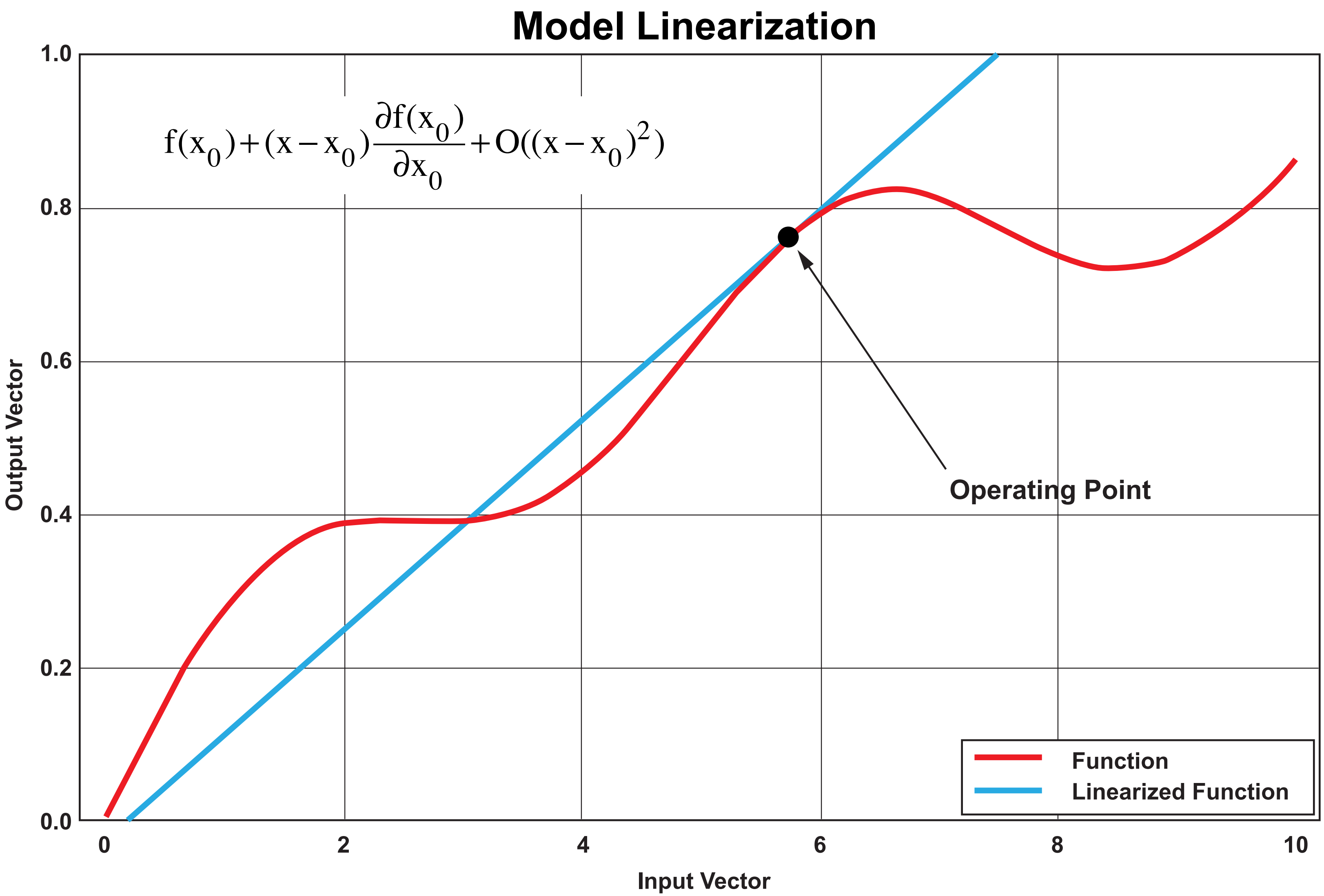
Spectral Considerations



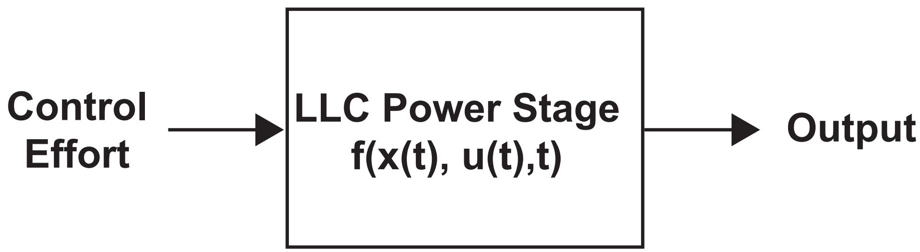


State Variable	Harmonic's Included
$I_{LR}(t)$	1, 3, 5, 7, 9, 11
$I_{LM}(t)$	1, 3, 5, 7
$V_{CR}(t)$	0, 1, 3, 5, 7
$V_{COUT}(t)$	0

Linearization



Describing Function Analysis



- **Linear System**
  - $\dot{x}(t) = A \cdot x(t) + B \cdot u(t)$
  - $y(t) = C \cdot x(t) + D \cdot u(t)$
- **Non-Linear System**
  - $\dot{x}(t) = f(x(t) + u(t), t)$
  - $y(t) = g(x(t) + u(t), t)$

$$x^{ss}(t) = X_0^{ss} + \sum_{k=1}^{\infty}\left(X_{ck}^{ss} \cdot \cos(k \cdot \omega_s \cdot t) + X_{ck}^{ss} \cdot \sin(k \cdot \omega_s \cdot t)\right)$$

$$F_{s_k}^{SS} = \frac{2}{T_s} \cdot \sum_{i=1}^{Q} \int_{T_i - 1}^{T_i} (A_i \cdot x(t)^{ss} + B_i \cdot U_0) \cdot \sin(k \cdot \omega_s \cdot t) \cdot dt$$

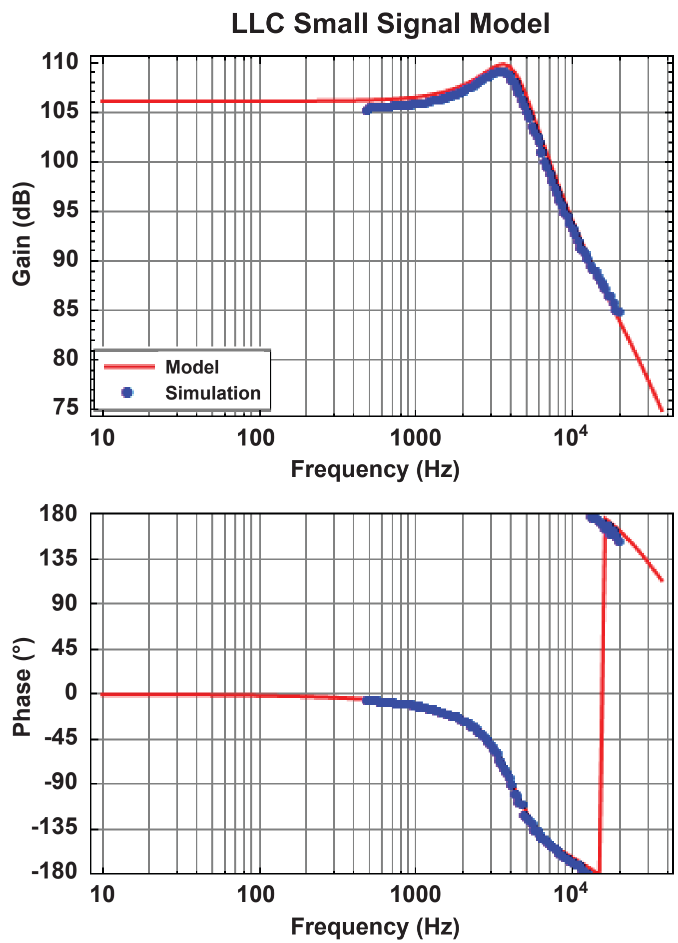
$$F_{c_k}^{SS} = \frac{2}{T_s} \cdot \sum_{i=1}^{Q} \int_{T_i - 1}^{T_i} (A_i \cdot x(t)^{ss} + B_i \cdot U_0) \cdot \cos(k \cdot \omega_s \cdot t) \cdot dt$$

Steady State Operating Point

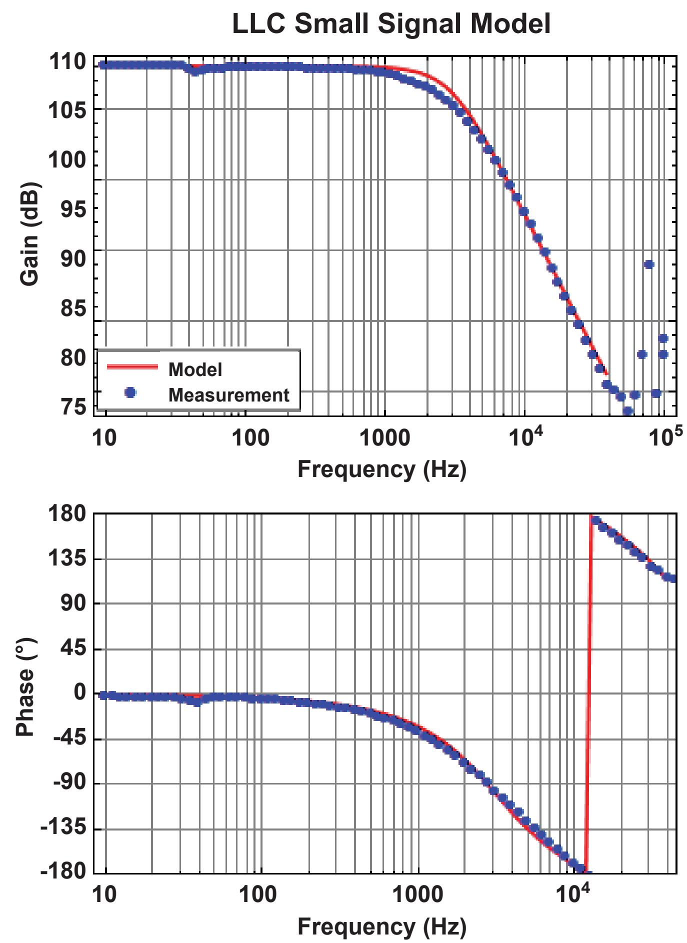
- **LLC + $C_{OUT} \Longrightarrow$ fourth order system**
- **Piecewise linear simulation:**
  - $\dot{x}_n(t) = A \cdot x_n(t) + B \cdot U$
  - $x_n(t_i) = (e^{A \cdot \Delta t} - I) \cdot A^{-1} \cdot B \cdot U + e^{A \cdot \Delta t} \cdot x_n(t_{i-1})$
- **Lightning fast, highly accurate results**

Model Validation

Model vs. Circuit Simulation



Model vs. Measurement

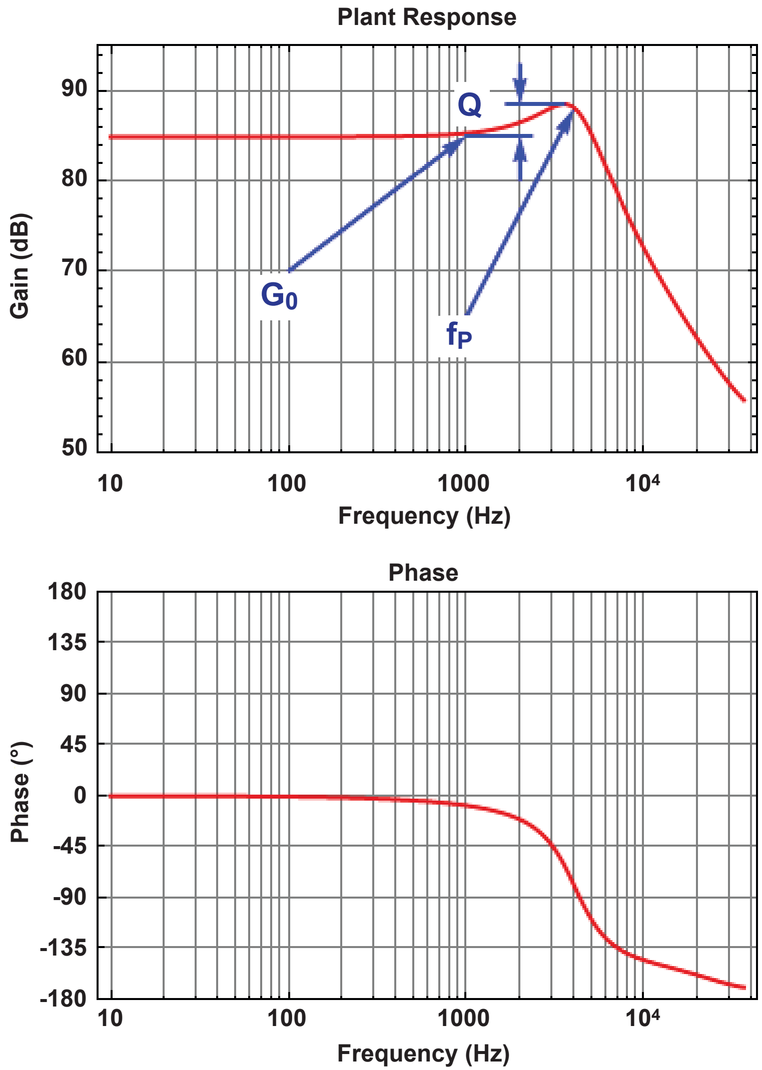


Compensation Objectives

- **Stability**
  - How do we achieve stability?
  - How do we ensure that we have sufficient stability margin for all operating points?
- **Performance**
  - Reference tracking
  - Load transient response
  - Input voltage transient rejection

Plant Analysis

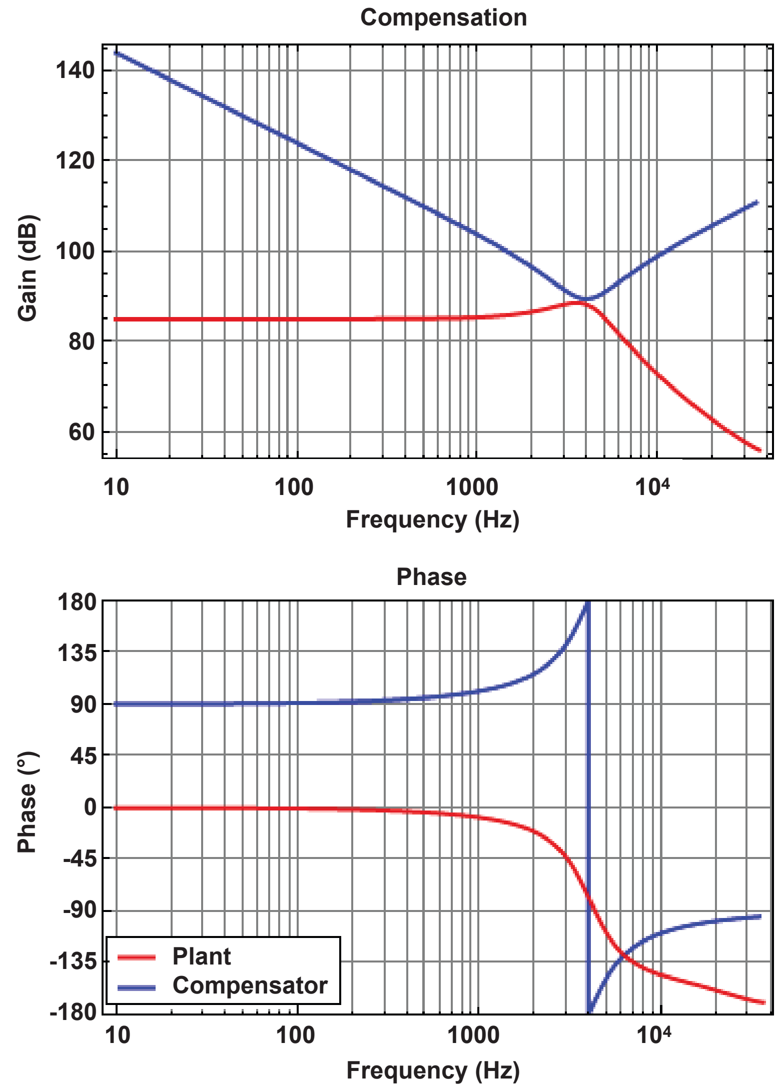
- 4^{TH} order system, 2^{ND} order response
  - $G_0 \sim 85$ dB
  - $Q \sim 1.35$
  - $f_p \sim 4$ kHz
- Stability Objectives
  - $\Phi_m \geq 45°$
  - $g_m \geq 10$ dB



Compensation

$$G_0 \cdot \frac{\left(\frac{s^2}{(2\cdot\pi\cdot f_Z)^2} + \frac{s}{2\cdot\pi\cdot f_Z\cdot Q_Z} + 1\right)}{s}$$

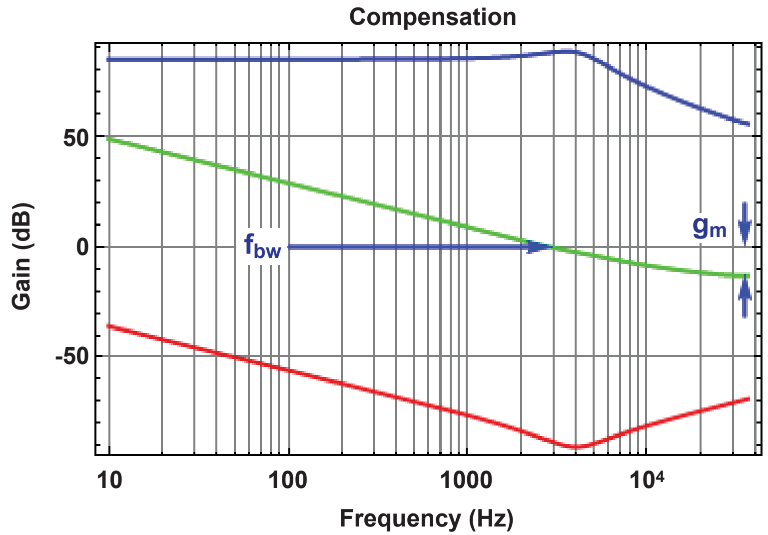
- 1/s term is required to eliminate DC error
- 2 zeros are required for stability
  - $Q_Z = 1.35$
  - $f_Z = 4$ kHz

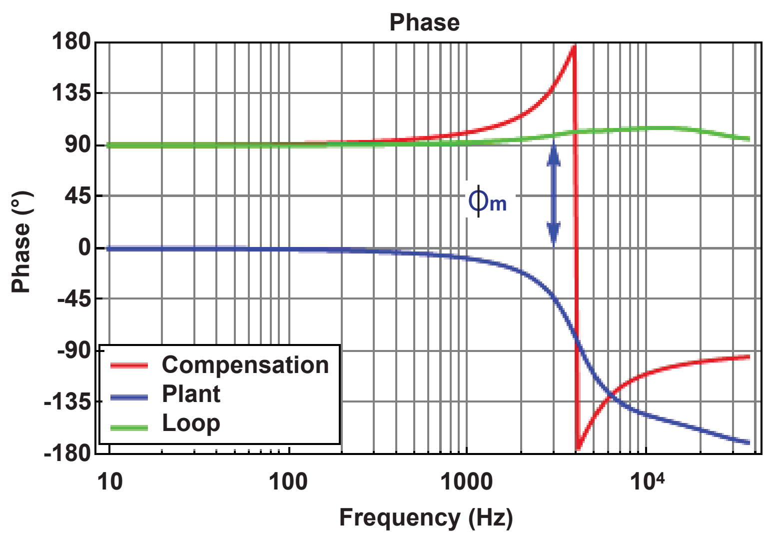


Overall Stability

- $$G_0 \cdot \frac{\left(\frac{s^2}{(2\cdot\pi\cdot f_Z)^2} + \frac{s}{2\cdot\pi\cdot f_Z \cdot Q_Z} + 1\right)}{s}$$
  - $G_0 = 0\ \text{dB}$
  - $Q_z = 1.35$
  - $f_z = 4\ \text{kHz}$

- Stability Margins
  - $\phi_m \cong 95°$
  - $g_m \cong 12\ \text{dB}$
  - $f_{bw} \cong 3\ \text{kHz}$



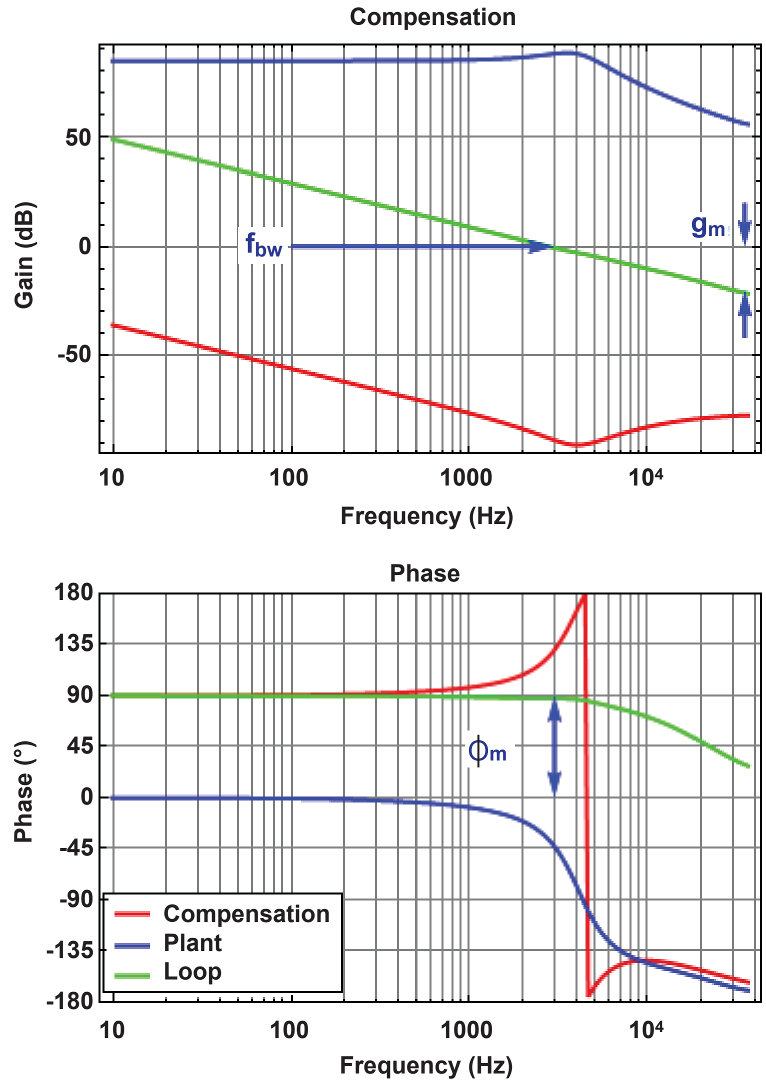


Overall Stability with an Extra Pole

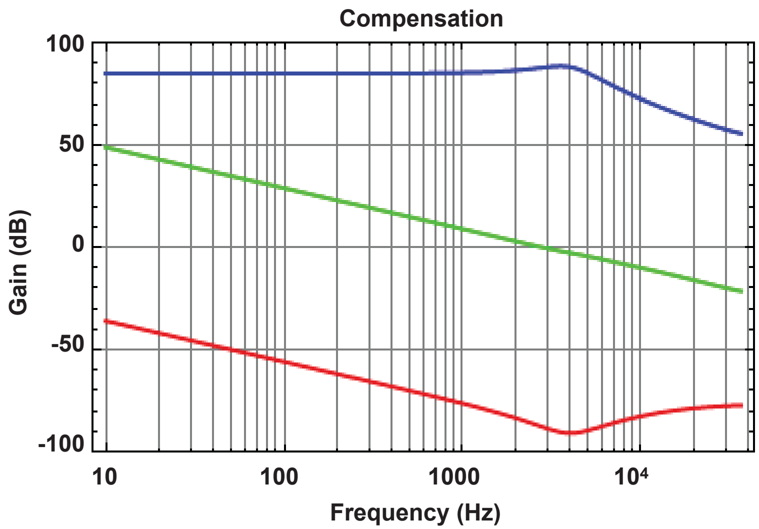
$$G_0 \cdot \frac{\left(\frac{s^2}{(2\cdot\pi\cdot f_Z)^2}+\frac{s}{2\cdot\pi\cdot f_Z\cdot Q_Z}+1\right)}{s\cdot\left(\frac{s}{2\cdot\pi\cdot f_p}+1\right)}$$

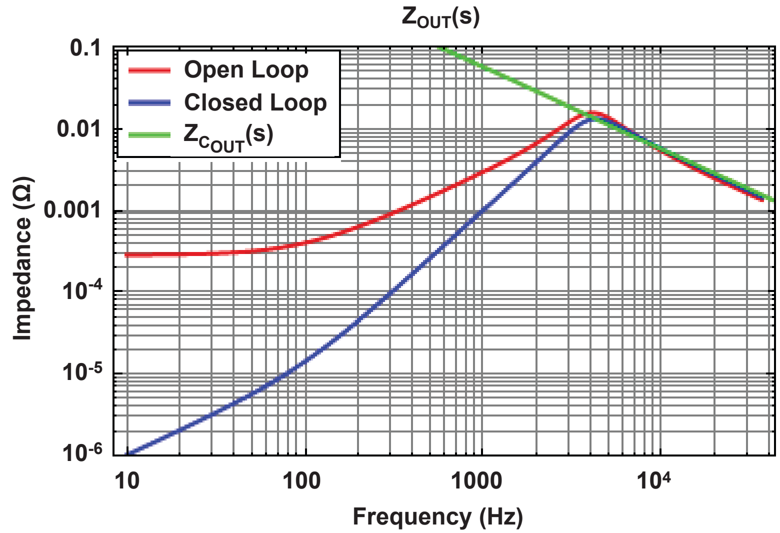
– $G_0 = 0\ \text{dB}$
– $Q_z = 1.35$
– $f_z = 4\ \text{kHz}$
– $f_p = 15\ \text{kHz}$

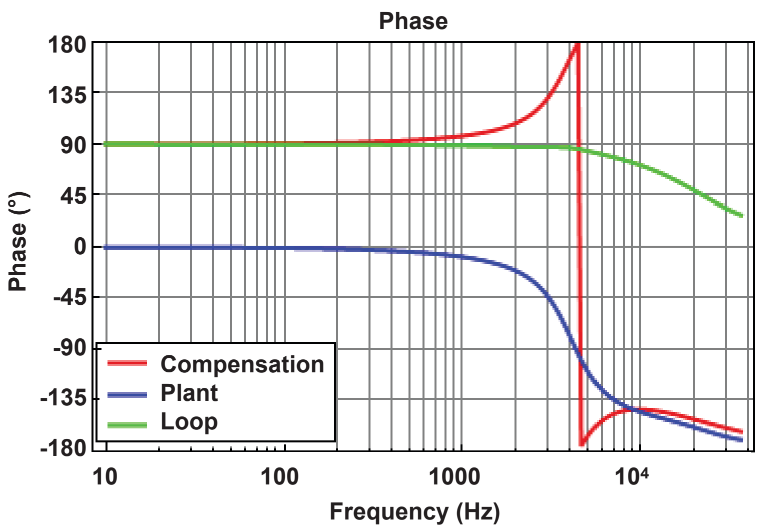
- Stability Margins
  – $\phi_m \cong 90°$
  – $g_m \cong 20\ \text{dB}$
  – $f_{bw} \cong 3\ \text{kHz}$

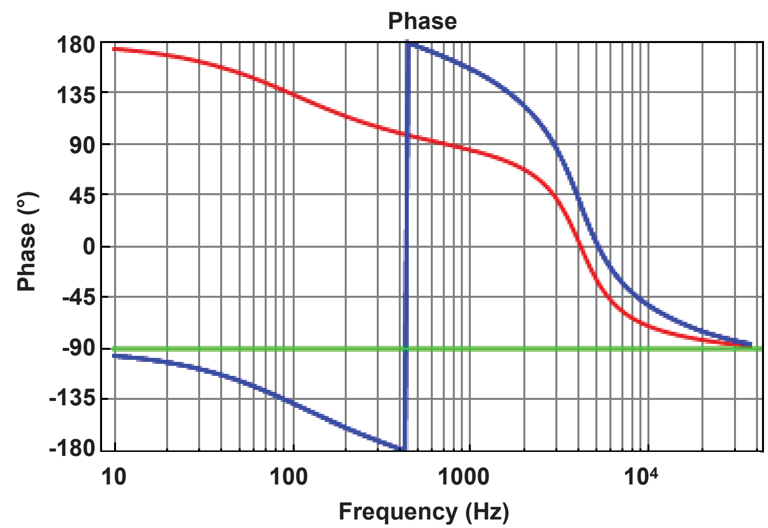


$Z_{OUT}(s)$, G_0 = 0 dB

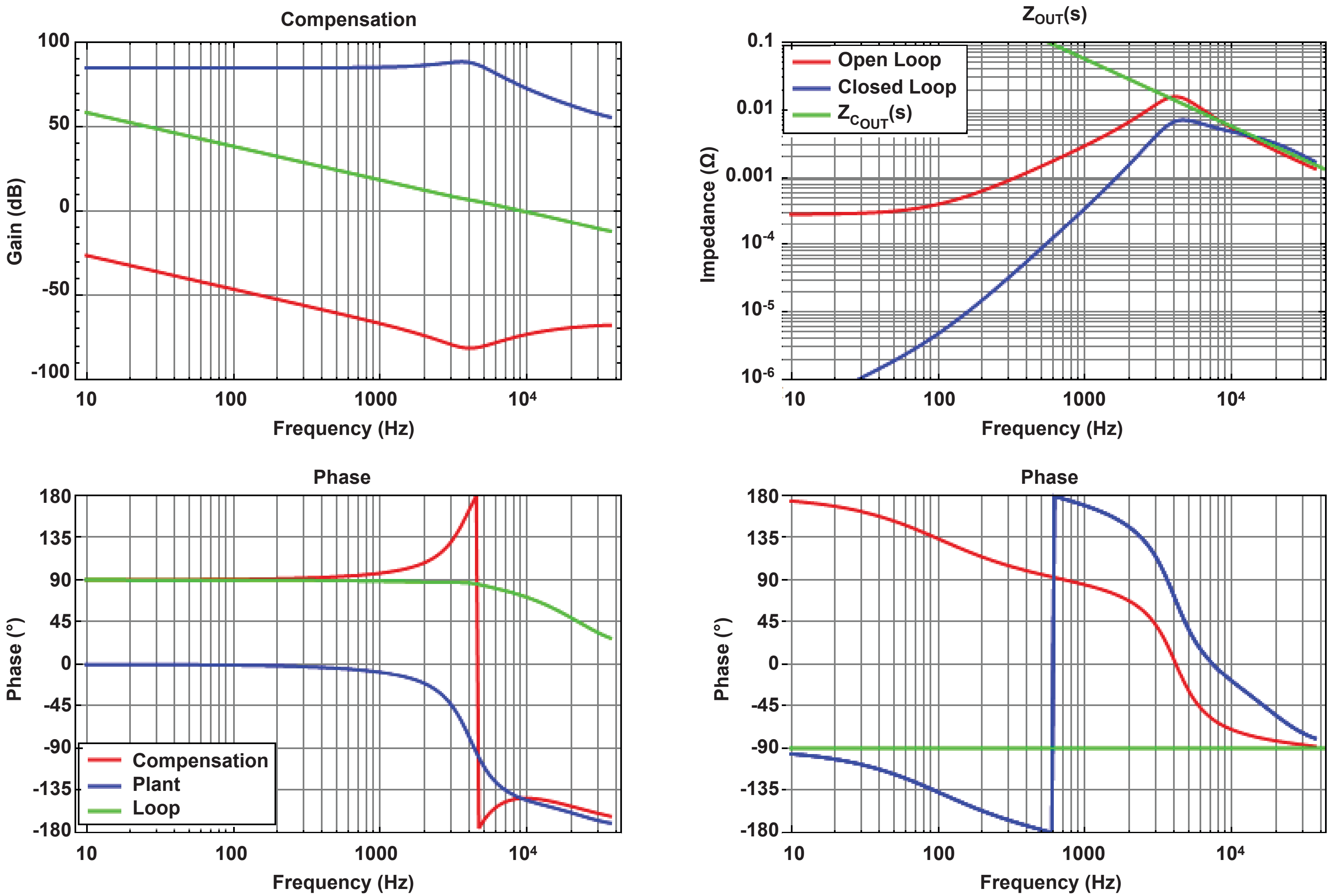

Compensation
100
50
0
-50
-100
Gain (dB)
10
100
1000
10⁴
Frequency (Hz)


ZOUT(s)
0.1
0.01
0.001
10-4
10-5
10-6
Impedance (Ω)
Open Loop
Closed Loop
ZCOUT(s)
10
100
1000
10⁴
Frequency (Hz)

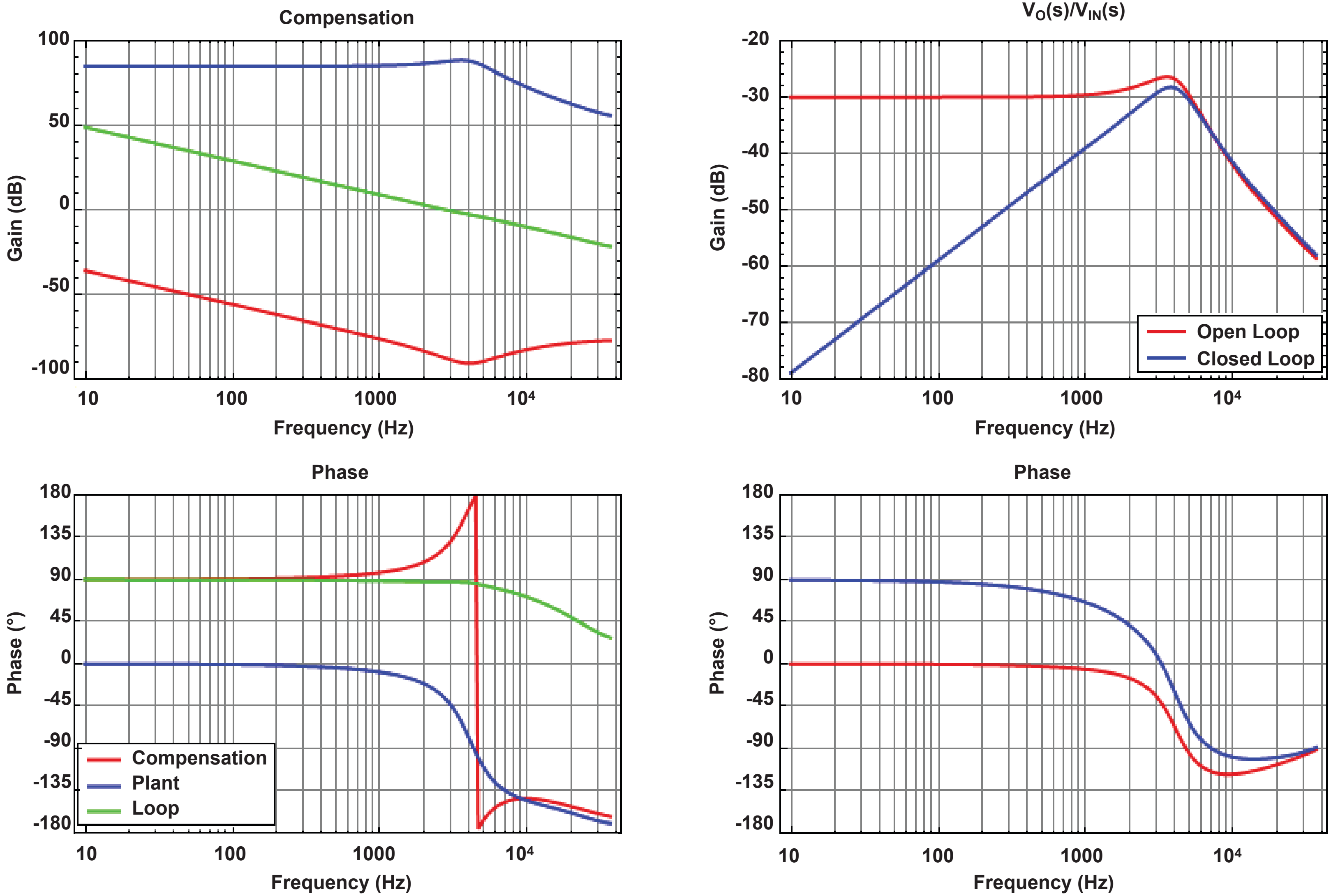

Phase
180
135
90
45
0
-45
-90
-135
-180
Phase (°)
Compensation
Plant
Loop
10
100
1000
10⁴
Frequency (Hz)


Phase
180
135
90
45
0
-45
-90
-135
-180
Phase (°)
10
100
1000
10⁴
Frequency (Hz)

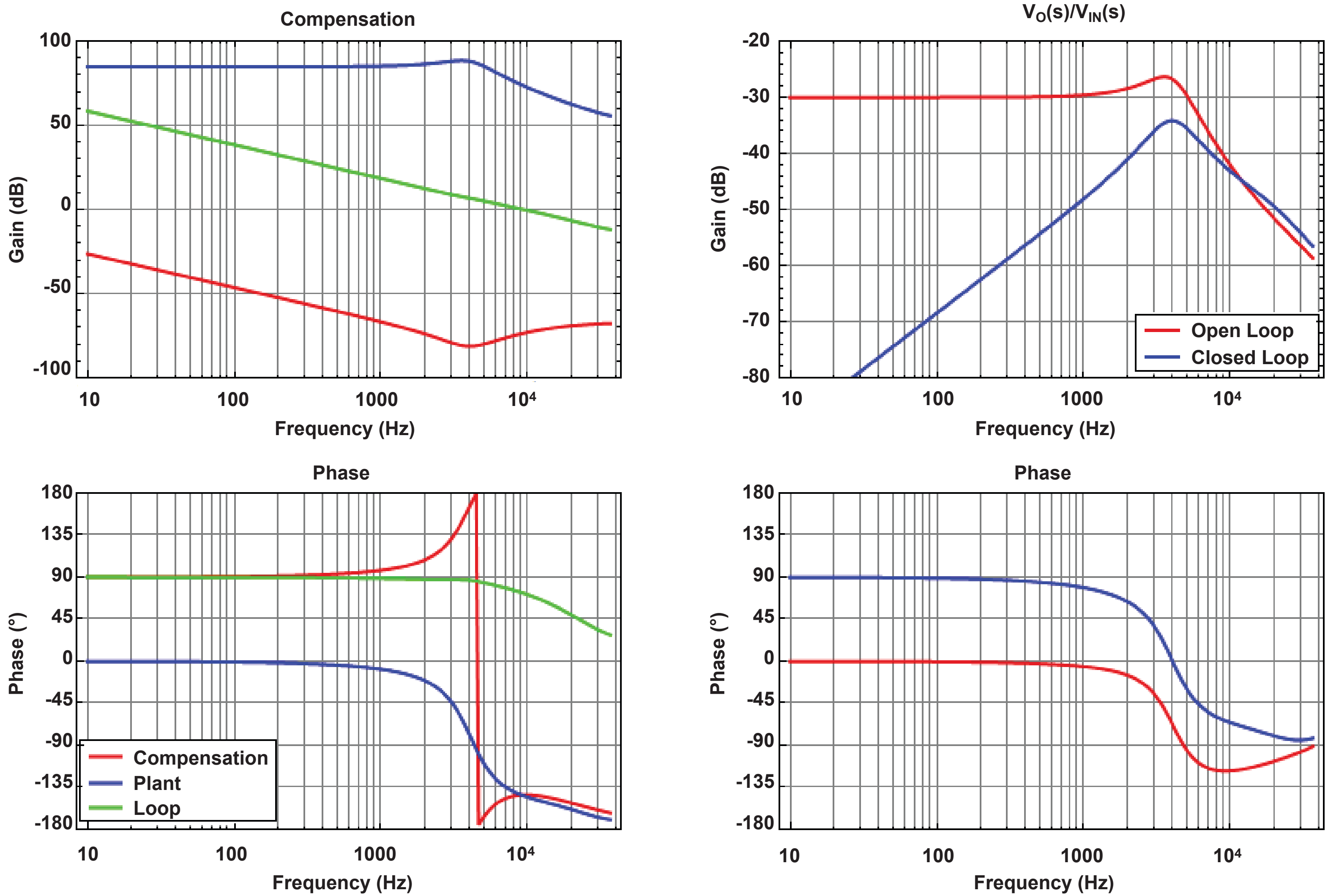
$Z_{OUT}(s)$, G_0 = 9.5 dB



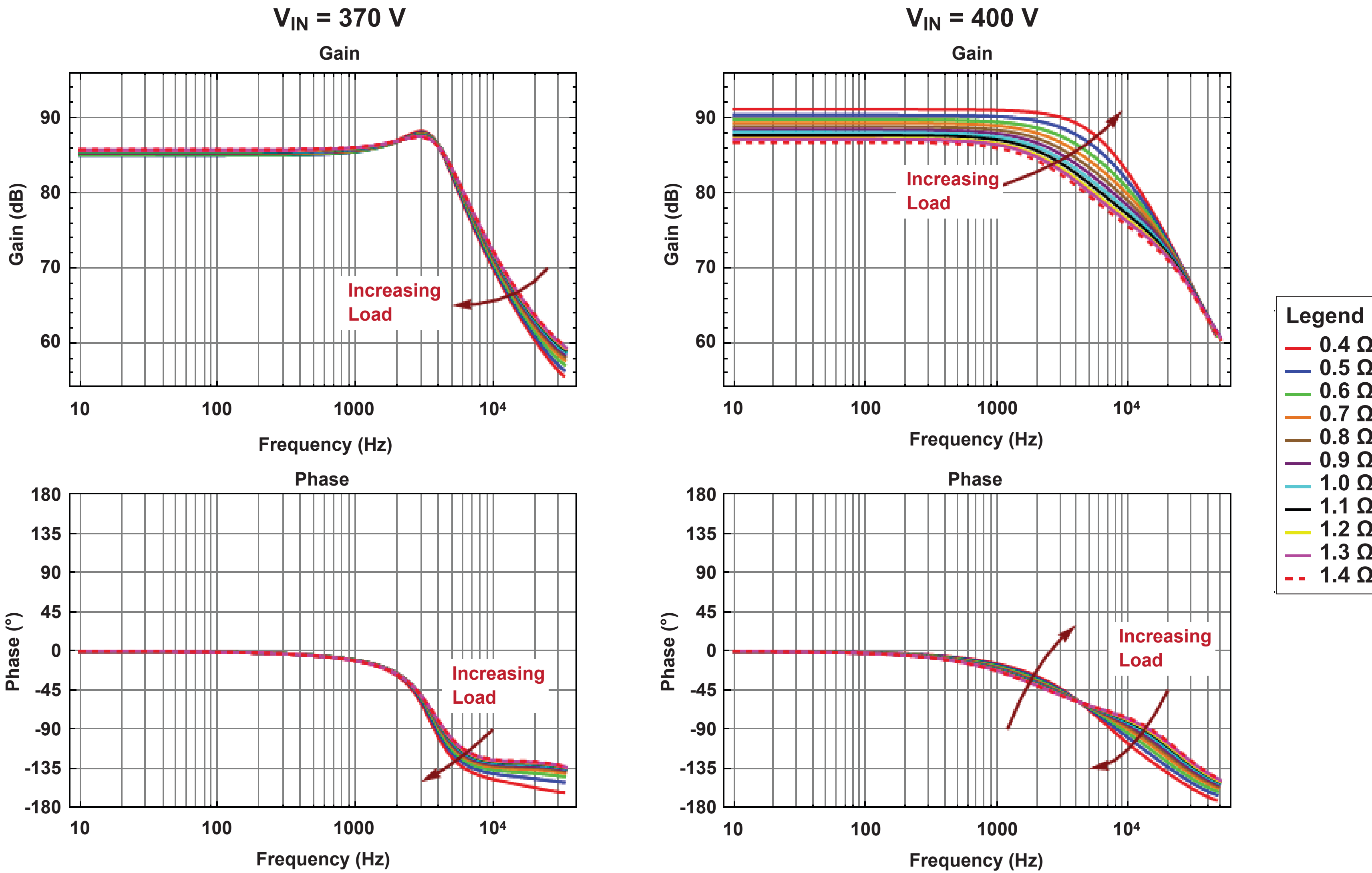
$\hat{V}_{OUT}(s)/\hat{V}_{IN}(s)$, G_0 = 0 dB



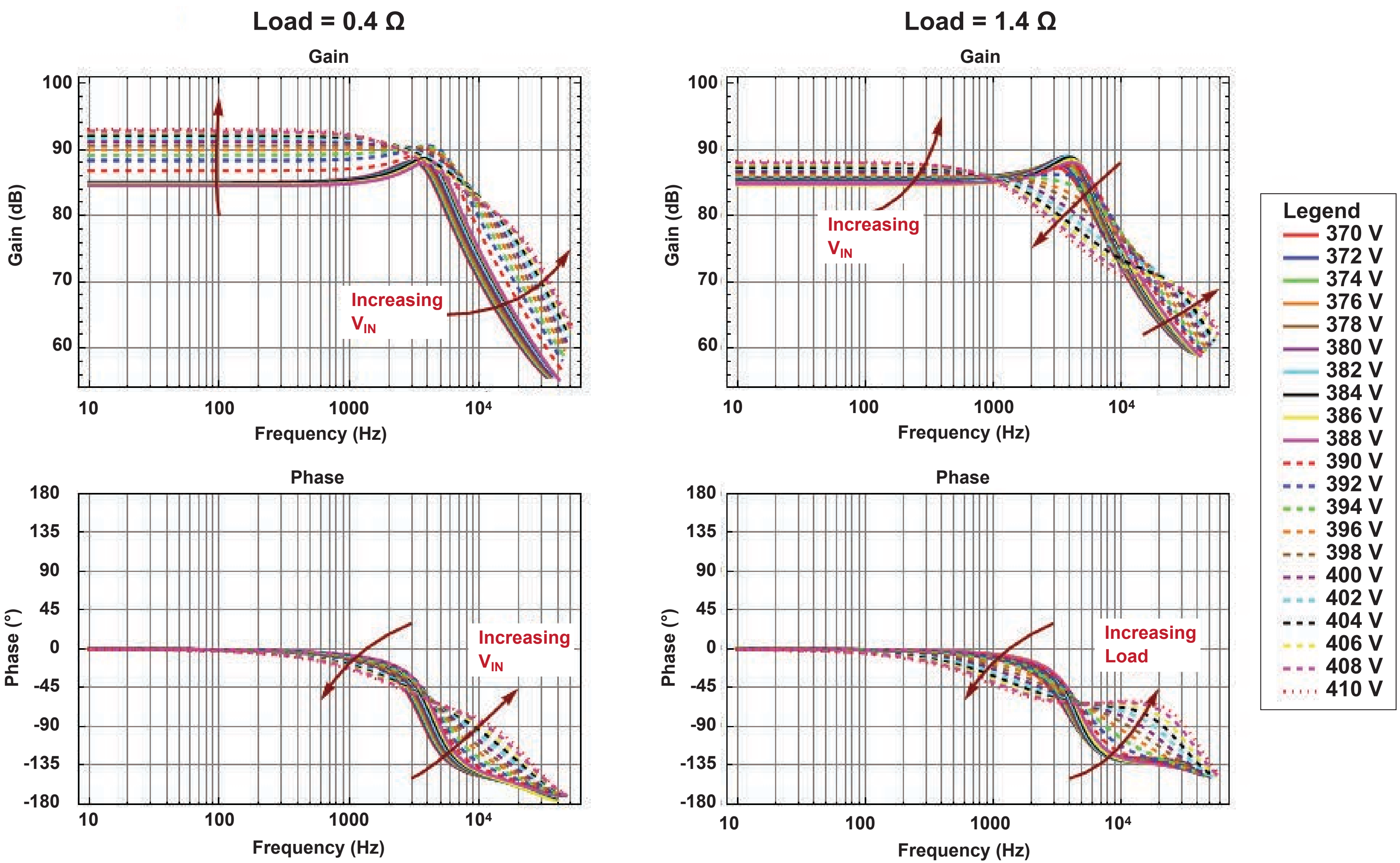
$\hat{V}_{OUT}(s)/\hat{V}_{IN}(s)$, G_0 = 9.5 dB



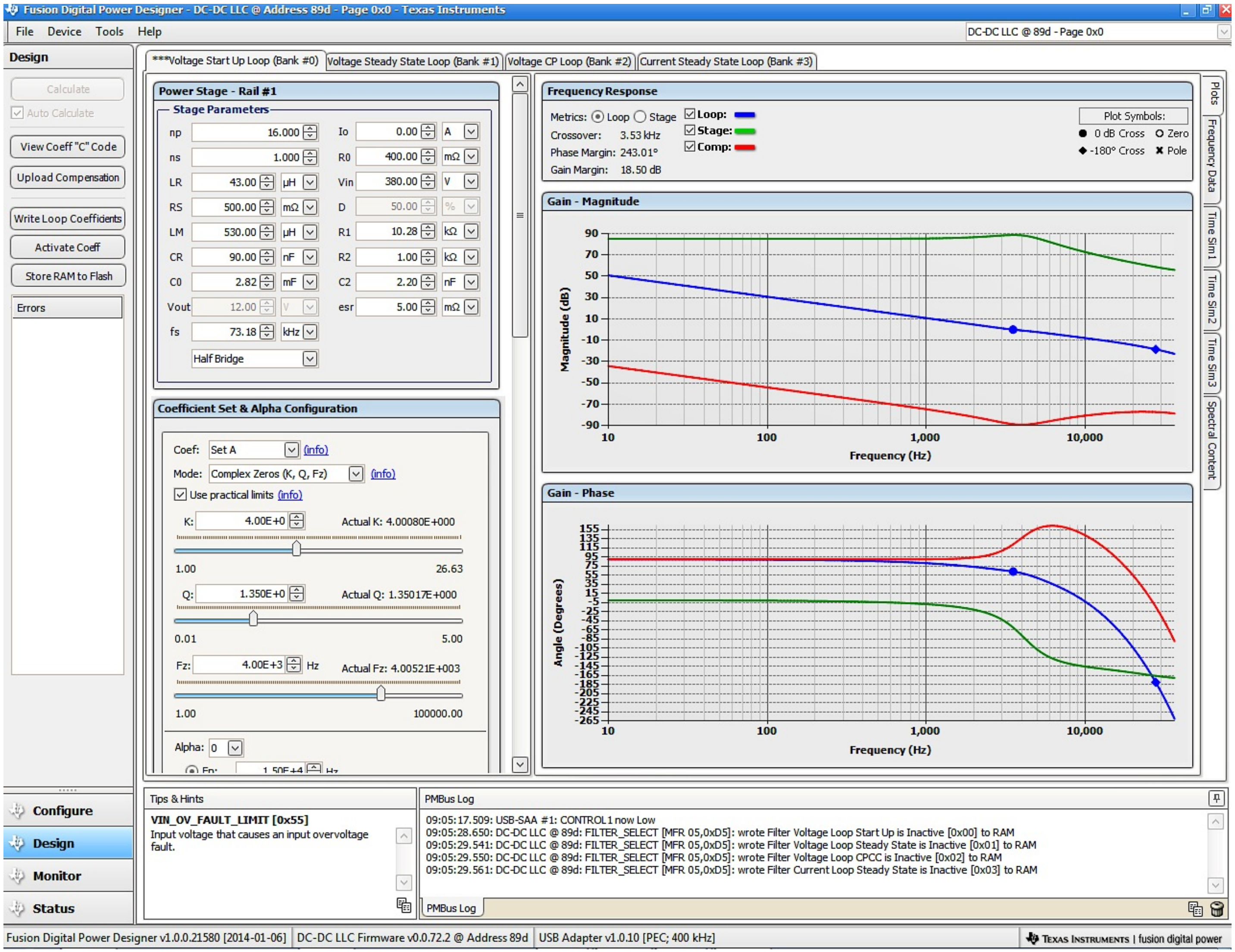
Load Variation



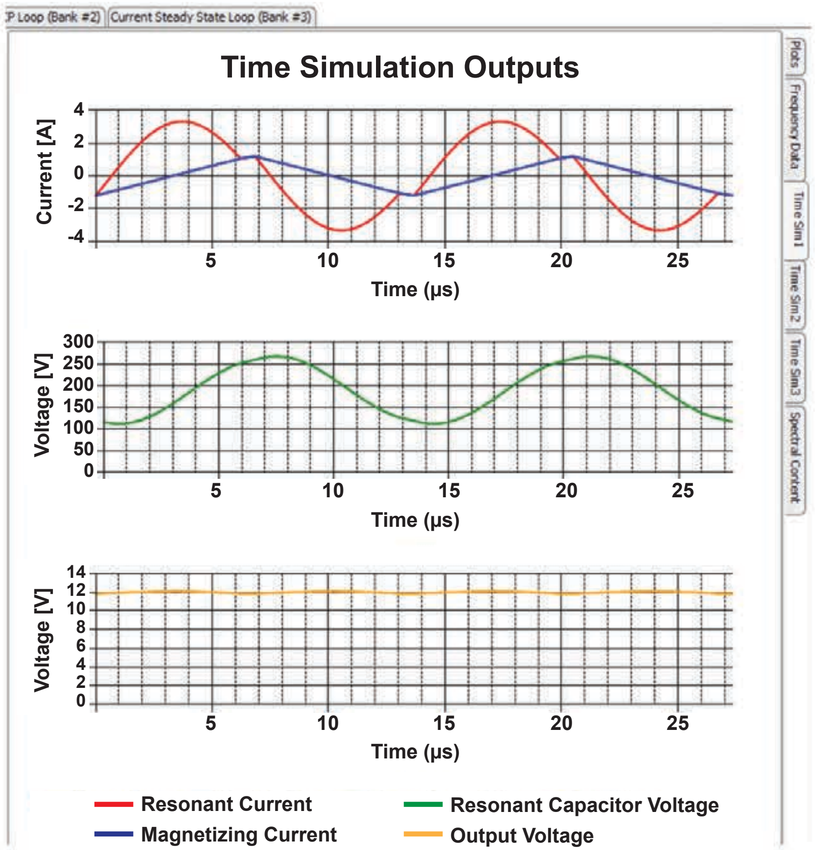
V_{IN} Variation

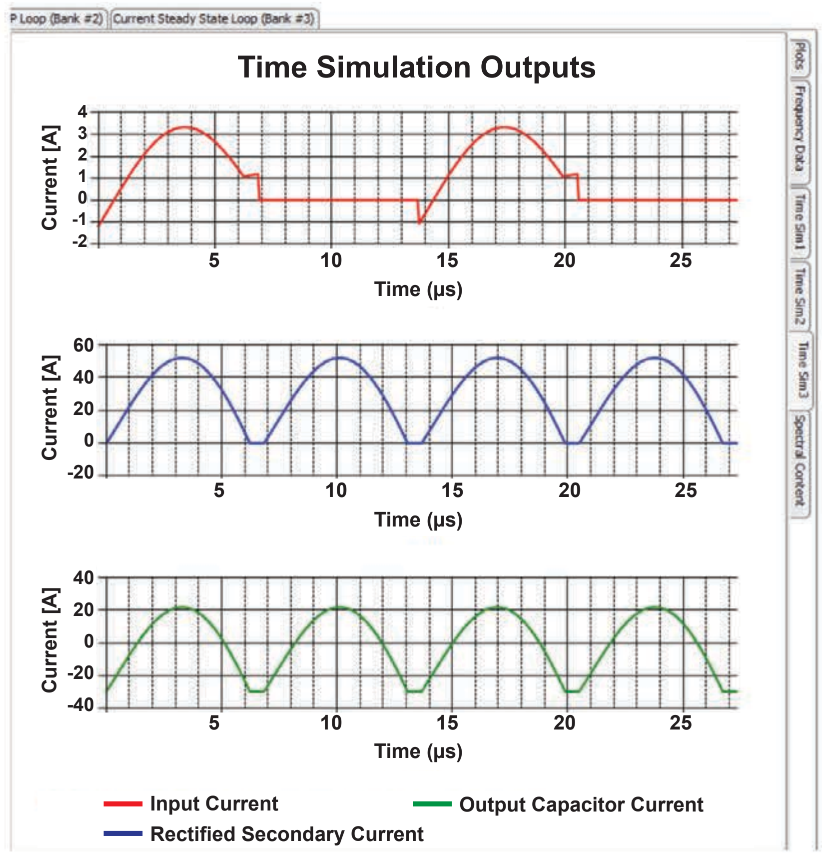


Fusion Digital Power Designer



Time Domain Behavior





Limitations

- Low efficiency scenarios may need additional work to achieve proper correlation
- Does not support PWM or PSM
- Corner cases may exist which limit the accuracy due to numerical approximations
- Additional work may be required to ensure accuracy, especially at higher frequencies

Conclusions

- Analytical predictions of plant pole zero behavior enables more robust compensation
  - Parameter variations
  - Extreme operating conditions
- Independent validation of the DC operating point
- Instant visualization of:
  - Key system waveforms
  - Harmonic content
- Seamless integration with TI standard isolated digital controllers

Future Work

- DC operating point
- Performance metrics
  - $Z_{OUT}(s)$
  - $Z_{IN}(s)$
  - $\widehat{V}_{OUT}(s)/\widehat{V}_{IN}(s)$
- Modulation methods
  - FM
  - PSM
  - PWM
- Additional States & Modes
  - Switching transitions
  - Body diode conduction